Unit 5

Linguistic Variables and Modifiers

Motivation

Our goal is to be able to proceed IF-THEN rules involving vague linguistic expressions which are modeled by fuzzy sets.

Question: *What is still missing?*

Linguistic variables establish the link between the linguistic expressions in the rules and the corresponding models (fuzzy sets).

Linguistic Variables

A *linguistic variable* is a quintuple of the form

$$V = (N, G, T, X, M),$$

where N, T, X, G, and M are defined as follows:

1. N is the name of the linguistic variable V
2. G is a grammar
3. T is the so-called *term set*, i.e. the set linguistic expressions resulting from G
4. X is the universe of discourse
5. M is a $T \to \mathcal{F}(X)$ mapping which defines the semantics—a fuzzy set on X—of each linguistic expression in T

Example 1

1. $N =$ "v1"
2. $G:$

$$\begin{array}{lcl} \bot & := & \langle\text{adjective}\rangle \; ; \\ \langle\text{adjective}\rangle & := & \text{“small”} \mid \text{“medium”} \mid \text{“large”} \; ; \end{array}$$

3. $T = \{\text{“small”}, \text{“medium”}, \text{“large”}\}$
4. $X = [0, 100]$
5. $M = \ldots$

Example 2

1. $N =$ "v2"

2. G :

$$
\begin{array}{lcl}
\perp & := & \langle\text{atomic}\rangle \ ; \\
\langle\text{atomic}\rangle & := & \langle\text{adjective}\rangle \mid \langle\text{adverb}\rangle\ \langle\text{adjective}\rangle \ ; \\
\langle\text{adjective}\rangle & := & \text{"small"} \mid \text{"medium"} \mid \text{"large"} \ ; \\
\langle\text{adverb}\rangle & := & \text{"at least"} \mid \text{"at most"} \ ;
\end{array}
$$

3. $$
\begin{aligned}
T = \{&\text{"small"}, \text{"medium"}, \text{"large"}, \\
&\text{"at least small"}, \text{"at least medium"}, \\
&\text{"at least large"}, \text{"at most small"}, \\
&\text{"at most medium"}, \text{"at most large"}\}
\end{aligned}
$$

4. $X = [0, 100]$

Example 3

1. $N =$ "v3"
2. G :

$\bot$	$:=$	⟨atomic⟩ | ⟨atomic⟩ ⟨binary⟩ ⟨atomic⟩ ;
⟨atomic⟩	$:=$	⟨adjective⟩ | ⟨adverb⟩ ⟨adjective⟩ ;
⟨adjective⟩	$:=$	"nb" | "nm" | "ns" | "z" | "ps"| "pm"| "pb";
⟨adverb⟩	$:=$	"at least" | "at most" ;
⟨binary⟩	$:=$	"and" | "or" ;

3. $T = \ldots$ (462 elements)
4. $X = [-100, 100]$
5. $M = \ldots$

Example 4

1. $N =$ “v4”
2. G :

$\perp$	:=	⟨exp⟩ ;
⟨exp⟩	:=	⟨atomic⟩ | “(” ⟨exp⟩ ⟨binary⟩ ⟨exp⟩ “)” | “(not” ⟨exp⟩ “)”;
⟨atomic⟩	:=	⟨adjective⟩ | ⟨adverb⟩ ⟨adjective⟩ | “between” ⟨adjective⟩ “and” ⟨adjective⟩ ;
⟨adjective⟩	:=	“nb” | “nm” | “ns” | “z” | “ps”| “pm”| “pb”;
⟨adverb⟩	:=	“at least” | “at most” ;
⟨binary⟩	:=	“and” | “or” ;

3. $T = \ldots$ (infinitely many elements)
4. $X = [-100, 100]$
5. $M = \ldots$

How Do We Define M?

- As long as T is finite (e.g. Examples 1 and 2), we can define $M(a)$ for each $a \in T$ separately
- If T is a big set (e.g. Example 3), this is cumbersome
- If T is infinite (e.g. Example 4), this is not possible anymore

Practically Feasible Way

- Define separate fuzzy sets $M(a)$ for all atomic adjectives a
- Use modifiers for adverbs
- Use fuzzy set operations for logical connectives

Unary Ordering-Based Modifiers

$$\mu_{M(\text{at least } a)}(x) = \sup\{\mu_{M(a)}(y) \mid y \leq x\}$$

$$\mu_{M(\text{at most } a)}(x) = \sup\{\mu_{M(a)}(y) \mid y \geq x\}$$

Note that, with the convention

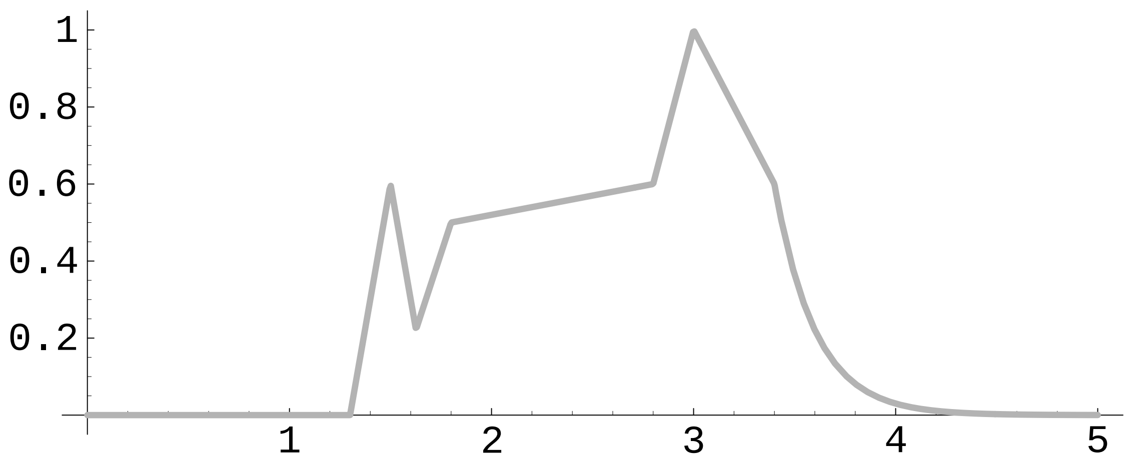
$$\mu_L(x, y) = \begin{cases} 1 & \text{if } x \leq y, \\ 0 & \text{if } x > y, \end{cases}$$

this means nothing else but the following:

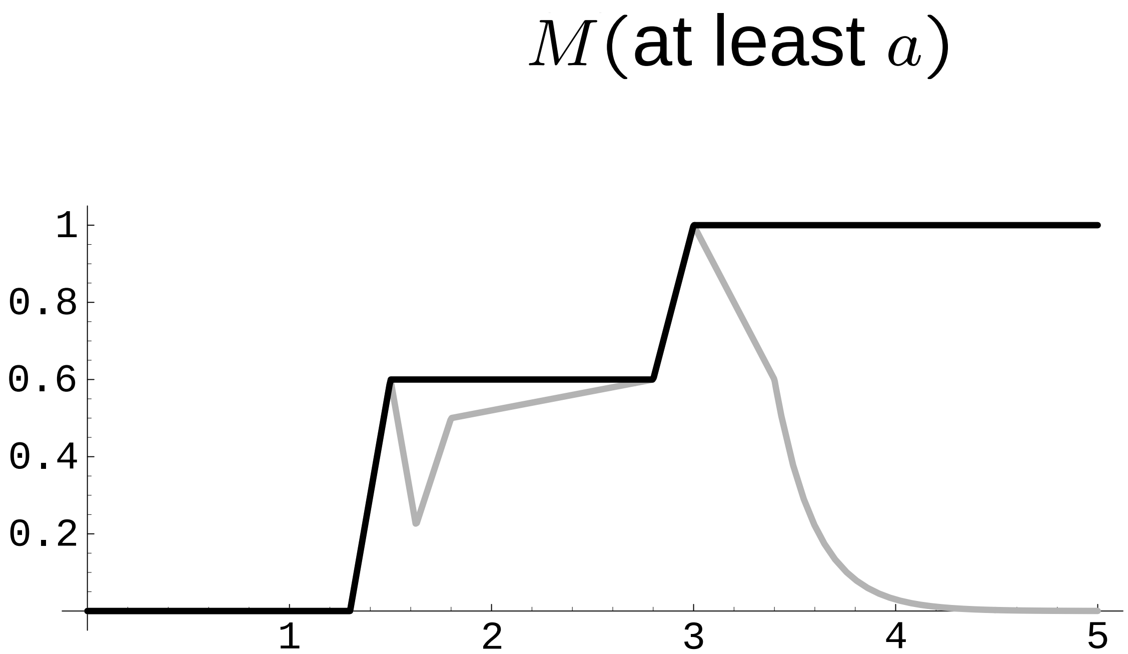
$$M(\text{at least } a) = L(M(a))$$

An Example

$$M(a)$$

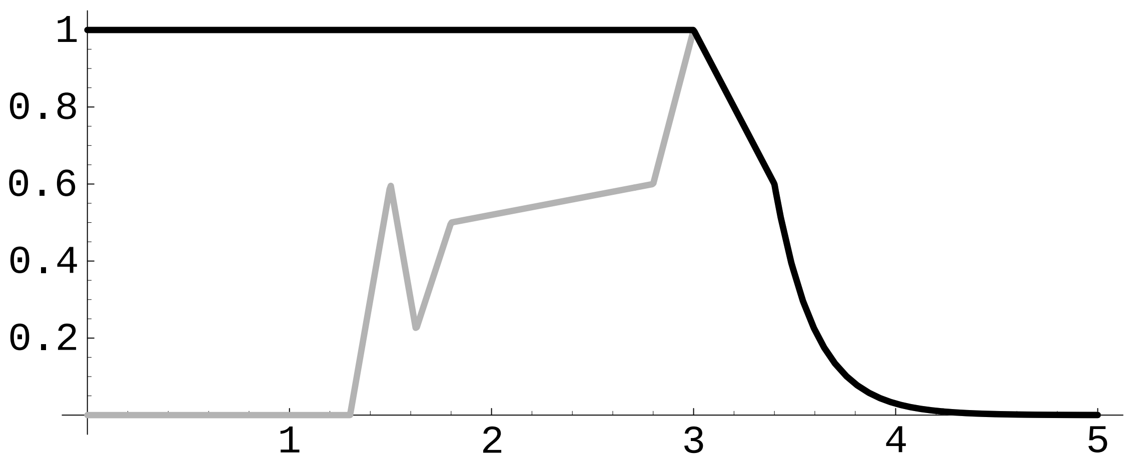


An Example



An Example

$M(\text{at most } a)$



Logical Connectives

Assume that (T, S, N) is a De Morgan triple. Then we can define the following:

$$\mu_{M(a \text{ and } b)}(x) = T\big(\mu_{M(a)}(x), \mu_{M(b)}(x)\big)$$
$$\mu_{M(a \text{ or } b)}(x) = S\big(\mu_{M(a)}(x), \mu_{M(b)}(x)\big)$$
$$\mu_{M(\text{not } a)}(x) = N\big(\mu_{M(a)}(x)\big)$$

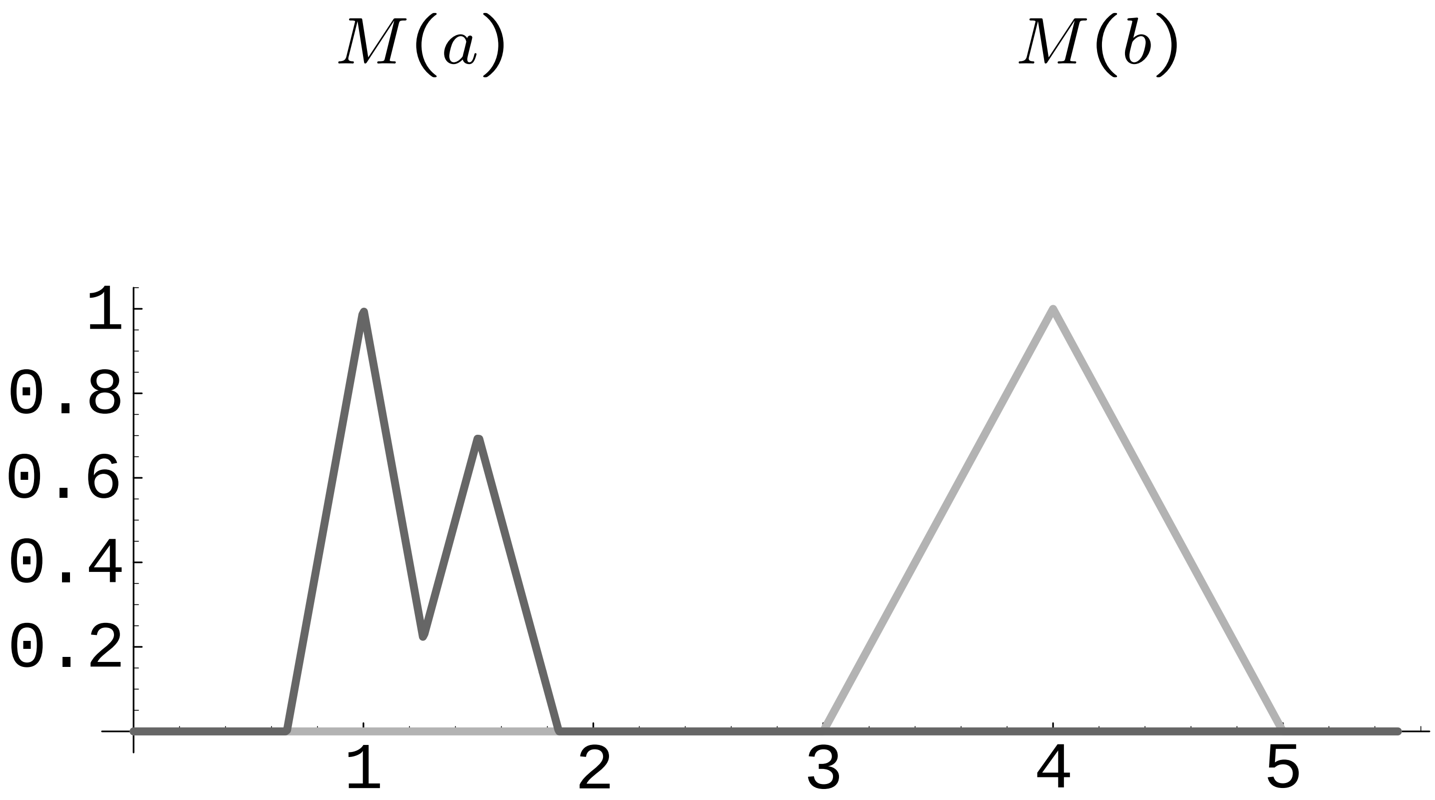
This means nothing else but the following:

$$M(a \text{ and } b) = M(a) \cap_T M(b)$$
$$M(a \text{ or } b) = M(a) \cup_S M(b)$$
$$M(\text{not } a) = \complement_N M(a)$$

The “Between” Modifier

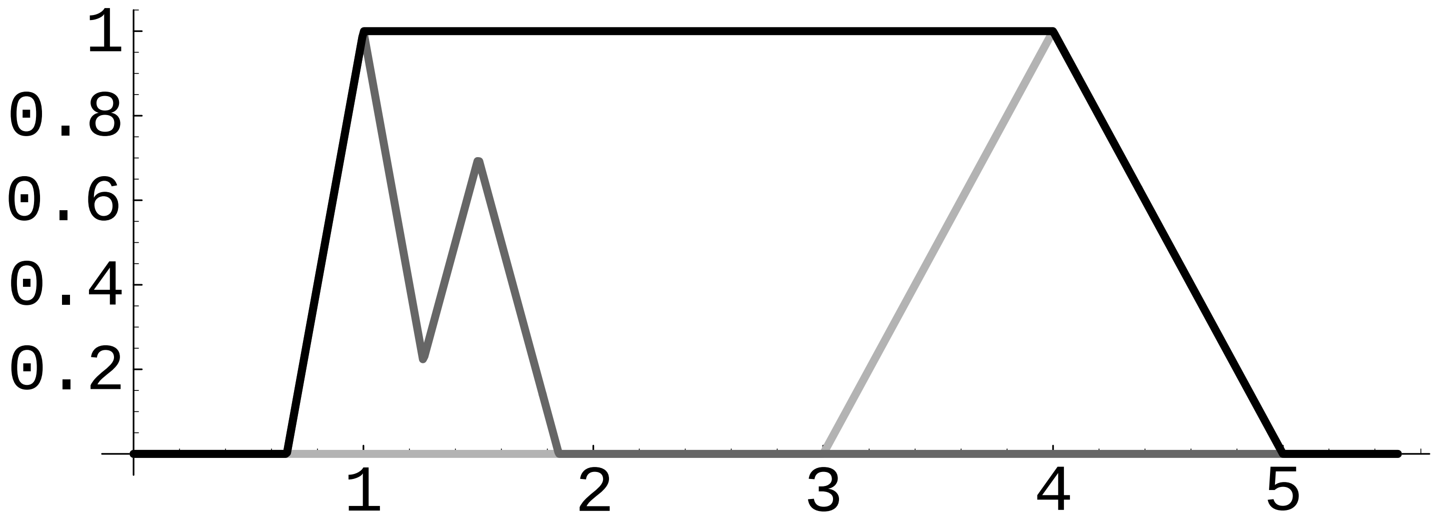
$$
\begin{aligned}
M\big(\text{between } a \text{ and } b\big) &= M\Big(\text{at least } (a \text{ or } b) \text{ and at most } (a \text{ or } b)\Big)\\
&= L\Big(M(a) \cup_S M(b)\Big) \cap_T L^{-1}\Big(M(a) \cup_S M(b)\Big)
\end{aligned}
$$

An Example



An Example

M(between a and b)



What About Other Adverbs?

- In principle, we can add any kind of adverb to the grammar G
- But how can we define the corresponding semantics?
- Again, either via separate fuzzy sets or via modifiers
- Notorious example: intensifying modifier “very” and weakening modifier “more or less”

Zadeh's Approach

$$\mu_{M(\text{very } a)}(x) = \left(\mu_{M(a)}(x)\right)^2$$

$$\mu_{M(\text{more or less } a)}(x) = \sqrt{\mu_{M(a)}(x)}$$

This approach is far too simplistic!

De Cock’s Approach

Suppose that R is a d-resemblance relation and T is a (left-)continuous t-norm.

$$M(\text{more or less } a) = R_T(M(a)),$$

$$M(\text{very } a) = R_T^{\bullet}(M(a)),$$

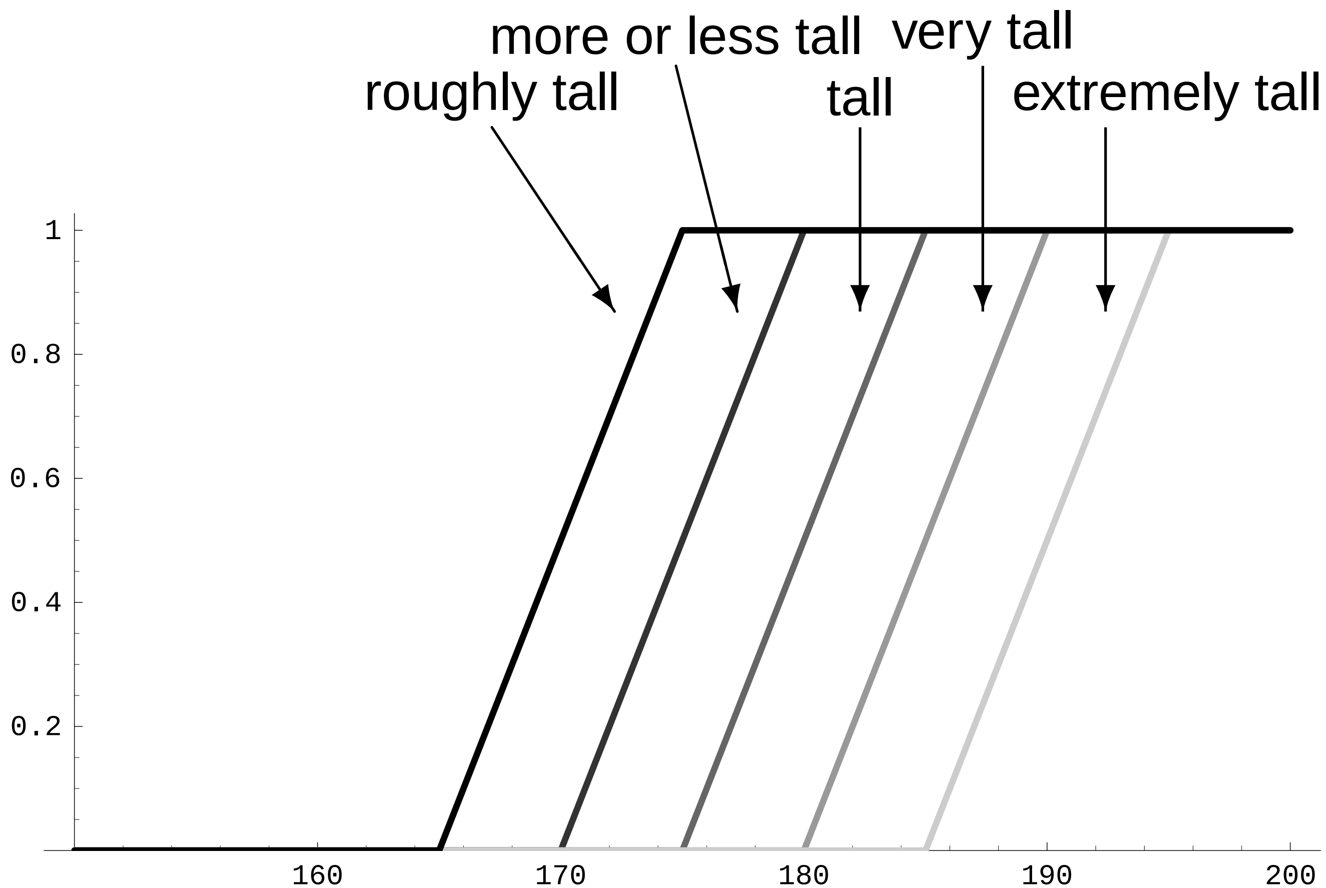
where $R_T^{\bullet}$ is a specific kind of image of R:

$$\mu_{R_T^{\bullet}(A)}(x) = \inf\{\overrightarrow{T}(\mu_R(x,y),\mu_A(y)) \mid y \in X\}$$

De Cock's Approach (cont'd)

$$
\begin{aligned}
M(\text{roughly } a) &= M(\text{more or less more or less } a)\\
&= R_T(R_T(M(a)))\\
M(\text{extremely } a) &= M(\text{very very } a)\\
&= R_T^{\bullet}(R_T^{\bullet}(M(a)))
\end{aligned}
$$

An Example


more or less tall
very tall
roughly tall
tall
extremely tall
1
0.8
0.6
0.4
0.2
160
170
180
190
200

Final Remarks

- There are several other approaches how to deal with “roughly”, “very”, “more or less”, etc.
- None of them is commonly accepted
- None of them is fully capable of capturing the subtle meanings of these adverbs in a satisfactory way