Aggregation Functions
Constructions, Characterizations,
and Functional Equations

Addendum:

Declarations of Contributions
Acceptance Letter
Remarks on Galley Proofs

Ergänzungen zur Habilitationsschrift

von
Susanne Saminger-Platz

Declarations of Contributions

List of co-authored articles (included in the thesis)

A02. S. Saminger-Platz, E.P. Klement, R. Mesiar. On extensions of triangular norms on bounded lattices. *Indagationes Mathematicae*, vol. 19, no. 1, pp. 135–150, 2008.

A03. S. Saminger-Platz, C. Sempi. A primer on triangle functions I. *Aequationes Mathematicae*, vol. 76, pp. 201–240, 2008.

A04. J.J. Quesada-Molina, S. Saminger-Platz, C. Sempi. Quasi-copulas with a given sub-diagonal section. *Nonlinear Analysis*, vol. 69, pp. 4654–4673, 2008.

A05. F. Durante, S. Saminger-Platz, P. Sarkoci. On representations of 2-increasing binary aggregation functions. *Information Sciences*, vol. 178, pp. 4634–4541, 2008.

A06. F. Durante, S. Saminger-Platz, P. Sarkoci. Rectangular patchwork for bivariate copulas and tail dependence. (accepted for publication in *Communications in Statistics — Theory and Methods*).

A07. S. Saminger, B. De Baets, H. De Meyer. On the dominance relation between ordinal sums of conjunctors. *Kybernetika*, vol. 42, no. 2, pp. 337–350, 2006.

A08. S. Saminger, P. Sarkoci, B. De Baets. The dominance relation on the class of continuous ordinal sum t-norms. In H.C.M. de Swart, E. Orłowska, M. Roubens, and G. Schmidt, editors, Theory and Applications of Relational Structures as Knowledge Instruments II, Springer, pp. 337–357, 2006.

A09. S. Saminger-Platz, B. De Baets, H. De Meyer. A generalization of the Mulholland inequality for continuous Archimedean t-norms. *Journal of Mathematical Analysis and Applications*, vol. 345, pp. 607–614, 2008.

A10. S. Saminger-Platz, B. De Baets, H. De Meyer. Differential inequality conditions for dominance between continuous Archimedean t-norms. (accepted for publication in *Mathematical Inequalities & Applications*).

A12. K. Maes, S. Saminger, B. De Baets. Representation and construction of self-dual aggregation operators. *European Journal of Operation Research*, vol. 177, pp. 472–487, 2007.

A13. S. Saminger-Platz, R. Mesiar, D. Dubois. Aggregation operators and commuting. *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1032–1045, 2007.

Declaration of Contribution of Authors to Paper

Susanne Saminger-Platz, Erich Peter Klement, Radko Mesiar
On extensions of triangular norms on bounded lattice.
Indagationes Mathematicae. (in press).

General comments

Inspired by the results and discussions about extensions of t-norms from subintervals to t-norms on bounded lattices (published in S. Saminger. *On ordinal sums of triangular norms on bounded lattices.* Fuzzy Sets and Systems, vol. 157, no. 10, pp. 1403–1416, 2006.), Susanne Saminger-Platz, Peter Klement and Radko Mesiar started in 2005 to investigate extensions of t-norms on sublattices rather than on subintervals. The discussions were carried out, whenever possible, in person or via email exchanging ideas and conjectures, proofs and counterexamples on different aspects of the joint work. Finally, in November 2005, during a visit of Radko Mesiar at JKU, a first manuscript could be finished and submitted to Indagationes Mathematicae in December 2005. This first version discussed strongest extensions of t-norms on sublattices resp. subintervals only, and provided additional information on special properties of t-norms and underlying lattices.

The reports on this first manuscript, asking for a revision, were received in Summer 2006. The contents of the manuscript, in particular Proposition 2.1, have been revised and improved during Susanne Saminger-Platz's sabbatical year at Lecce University. Moreover, all results on the weakest extension of t-norms on sublattices have been added in this period. The main responsibility for preparing this major revision, regards contents as well as from the editorial point of view, has been set to Susanne Saminger-Platz. The exchange of ideas for the weakest extensions has mainly been carried out between Susanne Saminger-Platz and Radko Mesiar.

Contributions by authors

- **Susanne Saminger-Platz** (50%)
  Revision of proof of Proposition 2.1, main ideas and results contained in Section 3, contribution to results in Section 4 and 5, most basic ideas and results contained in Section 6; participation in discussions; corresponding author.

- **Erich Peter Klement** (20%)
  Basic ideas and results contained in Section 2, 4, and 5; participation in discussion.

- **Radko Mesiar** (30%)
  Revision of proof of Proposition 2.1, basic ideas and results contained in Section 2, 4, and 5; contributions to results of Section 6; participation in discussion.

Susanne Saminger-Platz   Erich Peter Klement   Radko Mesiar

Declaration of Contribution of Authors to Paper

Susanne Saminger-Platz, Carlo Sempi
A primer on triangle functions I.
Aequationes Mathematicae. (in press).

At the very beginning of her postdoctoral stay in Lecce, Susanne Saminger-Platz and I immediately realized that there were many results on triangle functions scattered in the literature; however, in our opinion, what was missing was a unified and up-to-date survey, similar to what had been achieved, on a larger scale, by the book on t-norms by Klement, Mesiar and Pap. Therefore, we began discussing the possibility of writing such a survey. Thus began our cooperation on the two joint papers *A primer on triangle functions*.

Here I shall speak of the first one.

We had the opportunity, for about a year, of surveying the existing literature, of discussing almost daily, and at times, a few times a day, the selection and the order of topics to be included, of extending the known results. Whenever one of us wrote anything, this was read, commented, modified and, often, rewritten by the other one, or together during our joint sessions. For me, writing this paper has been a very pleasant and gratifying experience. Our cooperation has been so strict, that indeed I am at a loss in singling out the contribution of each one of us: every part of the paper has been rewritten, usually several times, by each of us. Therefore, the final version accepted by *Aequationes Math.* is truly a joint paper in the best sense of the word.

Carlo Sempi

Hereby, I declare to completely agree with Carlo Sempi's viewpoint.

Susanne Saminger-Platz

Declaration of Contribution of Authors to Paper

José Juan Quesada-Molina, Susanne Saminger-Platz, Carlo Sempi
Quasi-copulas with a given sub-diagonal section.
Nonlinear Analysis, Vol. 69, Issue 12, pp. 4654–4673, 2008.

General comments

The work on the contents of this paper had been initiated during a one-month-visit of José Juan Quesada-Molina at the University Lecce, in Autumn 2006, when Susanne Saminger-Platz spent her sabbatical year in the framework of an Erwin-Schrödinger Fellowship at the same university. First results have been achieved during this period by intensive collaboration and daily discussions, in particular several special cases of symmetric and non-symmetric (quasi-)copulas with a given sub-diagonal section could be proven. Section 7 of this paper still contains one of these symmetric copulas.

However, the intensive work already revealed that more general structures would build the basis of the considered class of functions. These aspects have finally been the W-ordinal sums of (quasi-)copulas (as outlined in Section 3), the splice technique (Section 4), the symmetrization of quasi-copulas (Section 5), as well as the determination of 2-increasing functions with given margins acting on a subsquare of the unit square (see also the articles *On representations of 2-increasing binary aggregation functions* and *Rectangular patchwork for bivariate copulas and tail dependence* both co-authored by F. Durante, S. Saminger-Platz, P. Sarkoci). Based on these considerations, additional results could be achieved and existing theorems proven in a new and more elegant way leading to a major rewriting of the original manuscript during Spring 2007 which was mainly carried out by Susanne Saminger-Platz.

After José Juan Quesada-Molina's departure from Lecce, all the communication of ideas, of preliminary and new results has been carried out on a more or less regular basis via email and discussions on Skype. Several partial drafts of the manuscript have been exchanged, commented, and revised by each of the co-authors. The last results could be finalized during a short visit of Susanne Saminger-Platz to Granada in April 2007. Therefore, the results contained in this contribution are the outcomes of joint work and collaboration in its best and original sense of meaning.

Susanne Saminger-Platz is the corresponding author. Careful proofreading of the manuscripts has been done by all co-authors. The authors are listed in alphabetical order.

Contributions by authors

- **José Juan Quesada-Molina** (30%)
  Basic ideas and proofs; participation in discussions.

- **Susanne Saminger-Platz** (40%)
  Basic ideas and proofs; participation in discussion; corresponding author.

- **Carlo Sempi** (30%)
  Basic ideas and proofs; participation in discussions.

José Juan Quesada-Molina   Susanne Saminger-Platz   Carlo Sempi

Declaration of Contribution of Authors to Papers

Fabrizio Durante, Susanne Saminger-Platz, Peter Sarkoci
(1) On representations of 2-increasing binary aggregation functions.
Information Sciences, Vol. 178, Issue 23, pp. 4534–4541, 2008,
and
(2) Rectangular patchwork for bivariate copulas and tail dependence.
Communications in Statistics (accepted).

General comments

During her investigations of quasi-copulas with a given sub-diagonal section, i.e., during her sabbatical year at the University Lecce, Susanne Saminger-Platz also studied 2-increasing functions with given margins acting on a subrectangle of the unit square. She succeeded to obtain a characterization of such functions by means of upper as well as upper and lower margins, further a determination of bounds for the class of 2-increasing function coinciding in all their margins.

At the same time, but completely independently of Susanne Saminger-Platz, also Fabrizio Durante together with Peter Sarkoci investigated these topics in Linz, achieving the same results. During a visit of Fabrizio Durante in Lecce the coincidence was recognized and the decision made to publish the results in a joint article. During the discussion, in particular after Susanne Saminger-Platz's return to Linz, it became clear that the results are also important for other fields, namely rectangular patchwork techniques for copulas, aspects of tail dependence and absolutely continuous copulas with given diagonal section. Moreover, the achieved results allow an additional perspective on copulas with a prescribed grid structure, or given horizontal and/or vertical sections, i.e., several constructions and bounds of classes of copulas. Therefore, the finally achieved results have been reassembled in two different articles for different audiences.

However and since still the main results, the characterizations, have been achieved independently by both "groups" of researchers, the amount of percentages, at least if provided such that they sum up to 100%, seem to be misleading for a true judgment of the scientific contribution of each co-author to the articles.

Susanne Saminger-Platz is the corresponding author of both articles. Careful proofreading of the final manuscript has been done by all co-authors. The authors are listed in alphabetical order.

Contributions by authors

(1) *On representations of 2-increasing binary aggregation functions*:

- **Fabrizio Durante** (33.3%)
  Basic ideas and proofs; participation in discussions.

- **Susanne Saminger-Platz** (33.3%)
  Basic ideas and proofs; participation in discussion; corresponding author.

- **Peter Sarkoci** (33.3%)
  Basic ideas and proofs; participation in discussions.

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(2) *Rectangular patchwork for bivariate copulas and tail dependence*:

- **Fabrizio Durante** (40%)
  Basic ideas and proofs; most basic ideas w.r.t. sampling, tail dependencies and absolutely continuous copulas with given diagonal section; participation in discussions.

- **Susanne Saminger-Platz** (30%)
  Basic ideas and proofs; participation in discussion; corresponding author.

- **Peter Sarkoci** (30%)
  Basic ideas and proofs; participation in discussions.

Fabrizio Durante   Susanne Saminger-Platz   Peter Sarkoci

Declaration of Contribution of Authors to Papers

Susanne Saminger-Platz, Bernard De Baets, Hans De Meyer
(1) On the dominance relation between ordinal sums of conjunctors.
Kybernetika, Vol. 42, No. 2, pp. 337–350, 2006,
(2) A generalization of the Mulholland inequality for continuous Archimedean t-norms.
Journal of Mathematical Analysis and Applications,
Vol. 345, pp. 607–614, 2008,
and
(3) Differential inequality conditions for dominance between continuous Archimedean t-norms.
Mathematical Inequalities & Applications (in press).

General comments

The long term cooperation on aspects of dominance started by a short term scientific mission of Susanne Saminger-Platz in the framework of the EU COST Action 274 (TARSKI: Theory and Applications of Relational Structure as Knowledge Instruments) to Gent University already in December 2003. The main goal of the investigations has been to lay bare the structure of dominance in the class of continuous t-norms as completely as possible. During that first month of collaboration, Susanne Saminger-Platz already could clarify most of the properties of dominance related to ordinal sum t-norms, as later on published in the article in Kybernetika. Hans De Meyer provided a first version of differential conditions, by this time just valid for strict t-norms. Further, basic steps towards the generalized Mulholland inequality, its relationship to dominance between continuous Archimedean t-norms, and the corresponding sufficient and necessary conditions could be made. All these investigations have been carried out by nearly daily discussions on the progress of individual achievements on all these aspects.

After Susanne Saminger-Platz's departure from Gent the research was constantly continued, however, rather on an individual basis with a more or less regular exchange of progress and new results via email resp. at conferences and meetings. In any case, achieved results have regularly been presented to an audience of interest. The generalized Mulholland inequality and especially its corresponding sufficient conditions turned out to be the most tricky part of the joint collaboration, at least for quite some period. Several approaches to solve the problem have been intensively discussed, also during a ten-days visit of Susanne Saminger-Platz to Gent University in October 2005. But only partial results and results on dominance between members of families of t-norms could be achieved. Finally, during her sabbatical year at Lecce University, Susanne Saminger-Platz found first proofs for the conditions as such also providing the theoretical basis for the validity of the results on differential conditions for dominance between strict as well as nilpotent t-norms.

Susanne Saminger-Platz is the corresponding author of all joint articles. Careful proofreading and modifications of the manuscript have been carried out by all co-authors.

Contributions by authors

(1) *On the dominance relation between ordinal sums of conjunctors*:

- **Susanne Saminger-Platz** (70%)
  Most basic ideas and results in Sections 4 and 5, basic ideas and result in Section 3; participation in discussion; corresponding author.
- **Bernard De Baets** (20%)
  Basic ideas and results, mainly in Section 3; participation in discussions.
- **Hans De Meyer** (10%)
  Basic ideas and results, mainly in Section 3; participation in discussions.

(2) *A generalization of the Mulholland inequality for continuous Archimedean t-norms*:

- **Susanne Saminger-Platz** (70%)
  Most basic ideas and main results; participation in discussion; corresponding author.
- **Bernard De Baets** (15%)
  Some basic ideas and results; participation in discussions.
- **Hans De Meyer** (15%)
  Some basic ideas and results; participation in discussions.

(3) *Differential inequality conditions for dominance between continuous Archimedean t-norms*:

- **Susanne Saminger-Platz** (40%)
  Basic ideas and results; participation in discussion; corresponding author.
- **Bernard De Baets** (30%)
  Basic ideas and results; participation in discussions.
- **Hans De Meyer** (30%)
  Basic ideas and results; participation in discussions.

Susanne Saminger-Platz   Bernard De Baets   Hans De Meyer

Declaration of Contribution of Authors to Papers

Susanne Saminger, Bernard De Baets, Peter Sarkoci
The dominance relation on the class of continuous t-norms from an ordinal sum point of view.
In H. C. M. de Swart, E. Orłowska, M. Roubens, and G. Schmidt, editors,
Theory and Applications of Relational Structures as Knowledge Instruments II,
Lecture Notes in Computer Science, pp. 337–357. Springer, 2006.

General comments

The main aim of this contribution has been to provide a self-contained overview on results on dominance between continuous ordinal sum t-norms which have been achieved within the framework of the EU COST Action 274 (TARSKI: Theory and Applications of Relational Structure as Knowledge Instruments). Both, Susanne Saminger-Platz as well as Peter Sarkoci have been on short term scientific missions to the University Gent, collaborating with Bernard De Baets on aspects of dominance. The results by Susanne Saminger-Platz on the isomorphism property of dominance and the dominance between ordinal sum t-norms laid the basis for the counter-example of dominance among all continuous t-norms obtained by Peter Sarkoci. Additionally, Peter Sarkoci has contributed a geometrical interpretation for dominance of some t-norm over the Łukasiewicz t-norm resp. the product t-norm.

Contributions by authors

- **Susanne Saminger** (40%)
  Most basic ideas and results in Sections 4, basic ideas and result in Sections 2.3 and 3.1; participation in discussions; main editorial work.

- **Peter Sarkoci** (40%)
  Most basic ideas and results in Sections 3.2 and 5; participation in discussions.

- **Bernard De Baets** (20%)
  Basic ideas and results; participation in discussions.

Susanne Saminger-Platz   Peter Sarkoci   Bernard De Baets

Declaration of Contribution of Authors to Paper

Koen C. Maes, Susanne Saminger, Bernard De Baets
Representation and construction of self-dual aggregation operators.
European Journal of Operational Research, Vol. 177, pp. 472–487, 2007 .

General comments

The joint investigations of self-dual aggregation functions had been initiated in Autumn 2004 during a short term scientific mission of Koen C. Maes to the Department of Knowledge-Based Mathematical Systems, Johannes Kepler University Linz, in the framework of the EU COST Action 274 (TARSKI: Theory and Applications of Relational Structure as Knowledge Instruments). By that time two characterization results for self-dual aggregation functions have been known and commonly used in literature. Susanne Saminger-Platz realized that both characterizations are based on the same principle, namely combining an aggregation function and its dual through another operation. Therefore, the investigation which operations would be admissible for serving as such a "combination rule" for the characterization of self-dual aggregation functions started. Soon after the first results have been achieved Koen C. Maes suggested to extend the joint work to equivalence classes of combination functions leading to the same self-dual aggregation function. The results achieved during the short term scientific mission focussed on self-dual aggregation function only, i.e., on N-invariant aggregation functions with N being the standard negation.

After his return to Gent, Koen C. Maes further elaborated, together with Bernard De Baets, the results and extended them to N-invariant aggregation functions with N an arbitrary standard negation. Exchange of ideas and progress of results has mainly been done via email.

Koen C. Maes is the corresponding author of this contribution. Careful proofreading of the manuscript has been done by all co-authors. The paper could be submitted by April 2005, was accepted in October 2005, available online February 2006, and finally published in 2007.

Contributions by authors

- **Koen C. Maes** (65%)
  Most basic ideas and proofs; participation in discussions; corresponding author.

- **Susanne Saminger** (20%)
  Some basic ideas and proofs; participation in discussion.

- **Bernard De Baets** (15%)
  Some basic ideas and proofs; participation in discussions.

Koen C. Maes   Susanne Saminger-Platz   Bernard De Baets

Declaration of Contribution of Authors to Paper

Susanne Saminger-Platz, Radko Mesiar, Didier Dubois
Aggregation Operators and Commuting.
IEEE Transactions of Fuzzy Systems, Vol. 15, No. 6, pp. 1032–1045.

General comments

Inspired by the property of dominance, a functional inequality between two operations, soon the idea for investigating the corresponding functional equation arose. Susanne Saminger-Platz and Radko Mesiar started to collaborate on this topic. The full characterization of all operators commuting with bisymmetric aggregation operators with some neutral element (see Proposition 21) revealed that the class of t-norms, t-conorms and uninorms play a special role. In particular uninorms, due to the fact that their neutral element is an interior element of their domain, have been appealing because of their role in bipolar decision making. Didier Dubois who is an expert in decision making and the recent developments on bipolarity in this field joint the team and participated and contributed to many fruitful discussion.

The work on this paper has been carried out in intensive discussion, in person at some conferences and meetings, as well as by email. Therefore, the results of the joint paper are indeed the fruits of a joint collaboration rather than a union of typed lines containing some results. Susanne Saminger-Platz is the corresponding author of this article.

Contributions by authors

- **Susanne Saminger-Platz** (70%)
  Majority of ideas and results of sections III, V, VI, some results of section IV; participation in discussions; corresponding author.

- **Radko Mesiar** (20%)
  Most basic ideas and results of section IV, some basic ideas and results of section III, V, VI; participation in discussions.

- **Didier Dubois** (10%)
  Participation in discussions, mainly on relevant aspects of (bipolar) decision making.

Susanne Saminger-Platz   Radko Mesiar   Didier Dubois

Acceptance Letter

Acceptance letter

for
A06. Rectangular patchwork for bivariate copulas and tail dependence
Communication in Statistics — Theory and Methods

(communicated by e-mail)

21-Oct-2008

Dear Dr. Saminger-Platz:

Ref: Rectangular patchwork for bivariate copulas and tail dependence

Our referees have now considered your paper and have recommended publication in Communications in Statistics Theory and Methods. We are pleased to accept your paper in its current form which will now be forwarded to the publisher for copy editing and typesetting.

You will receive proofs for checking, and instructions for transfer of copyright in due course.

The publisher also requests that proofs are checked and returned within 48 hours of receipt.

Thank you for your contribution to Communications in Statistics – Theory and Methods and we look forward to receiving further submissions from you.

Sincerely,
N. Balakrishnan
Editor in Chief,
Communications in Statistics Theory and Methods
comstat@mcmaster.ca

Remarks on Galley Proofs

JOHANNES KEPLER
UNIVERSITÄT LINZ
Netzwerk für Forschung, Lehre und Praxis

**Institut für Wissensbasierte
Mathematische Systeme**

Fuzzy Logic Laboratorium

Univ.Ass. [in] Dr.[in] Susanne Saminger-Platz
Tel.: +43/70/2468-9195
Fax: +43/70/2468-1351
susanne.saminger-platz@jku.at

Sekretariat:
Sabine Lumpi / DW 9194
sabine.lumpi@jku.at

Remarks on the galley proofs of "Differential inequality conditions for dominance between continuous Archimedean t-norms",
coauthored by Susanne Saminger-Platz, Bernard De Baets, and Hans De Meyer,
accepted for publication in Mathematical Inequalities & Applications

Dear editors,

The galley proofs of our article are already very nicely set. A few minor changes shall still be applied which we briefly summarize:

- The MSC(2000) should be: 39B62, 26D97, 26B35

- Last line of the abstract: please change to: parametric families of t-norms.

- Please change "aggregation operators" to "aggregation functions" on page 1, last line of the first paragraph of the introduction, as well as on page 2 line -1 of the page.

- On page 2, display of T_D: replace ";" by "," at the end of the first line of the display.

- On page 3, last line: add a comma at the end

- On page 5, Proposition 3, line 2: Please insert the linebreak either after "h=" or after the colon in the next line, i.e., keep together "h= t_1 o $t_2^{(-1)}$:".

- On page 5, second last paragraph, line 5: change "a same parametric family" to "a parametric family".

- On page 6, in formulas (3)and (4) replace ";" by ",".

- On page 7: line -4: end the display with ",".

- On page 10: line 5 of first paragraph: change to "is in accordance …".

- On page 10: display of t^{SS} in the middle of the page: drop ',' at the end of the display.

- On page 10, third paragraph from below: Avoid the linebreak in the word "t-norms".

- On page 12, display of t_\lambda^SW: the comma at the end of the second line should be dropped.

- On page 12, display with t', t'' and t''': at the end of the line with t''', replace ";" by ".", and let start the next line with a capital: "In case ...'"
- Many of the equivalent inequalities are aligned w.r.t. to both, the inequality sign as well as the equivalence symbol "$\Leftrightarrow$". In many cases this has been successfully implemented in the galley proofs but for a few cases where it still should be adopted, namely on page 11 last displays, on page 13 displays on the top of the page, on page 16 last displays, and finally on page 17 last displays.
- On page 15: Corollary 14: add "," at the end of line (ii).
- On page 16, paragraph in the middle of the page, there is a typo: it should be "t_1^D" rather than "t_1^B". Two lines later it should be "the generalized Mulholland inequality …" rather than "the Mulholland inequality".
- In Reference [1] drop "." after BODENHOFER.
- In Reference [4] drop "," before AND.
- In Reference [12] there is a superfluous "n" at the end of my name
- The names of my coauthors are correctly written as "Bernard De Baets" and "Hans De Meyer" (both times "De" is written with a capital letter at the beginning).

Kind regards,
Susanne Saminger-Platz